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THE CONTACT PERFORMANCE OF A STANDARD LINEAR SOLID HALF-SPACE UNDER MONOTONIC LOADING

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Abstract: Proficient design of machine elements requires knowledge of contact stresses and deformation, especially when the contact is concentrated, and high stress gradients arise in confined parts of the bulk. Even with time-independent material properties, a general contact solution can only be numerical and relies on spatial discretisation. Consideration of viscoelasticity as a constitutive law for the contacting materials introduces the time parameter in the problem model. The numerical approach brings the advantage of numerical integration that can be performed over both space and time dimensions. The base for the application of superposition of effects to linear viscoelasticity stems from the Boltzmann superposition principle and the Boltzmann-Volterra integral equation. The latter, combined with Green's functions for a semi-infinite isotropic continuum, facilitates the evaluation of the displacement exhibited by a viscoelastic flat under arbitrary pressure. By considering the contacting bodies as half-spaces, which is standard for concentrated contacts, a numerical solution to the viscoelastic contact problem can be achieved. Within the developed computational procedure, time-dependent material response is incorporated via a creep compliance function, formulated in alignment with classical linear viscoelastic frameworks, such as the Maxwell or Kelvin-Voigt constitutive models. In this paper, a more complex Standard Linear Solid model is employed because it is the simplest unit that can replicate creep, recovery, and stress relaxation. The assembled algorithm was implemented in MATLAB by reusing code previously developed by the same author for elastic contact. The simulations targeted scenarios with monotonic loading: step, ramped, and step followed by ramped down loading. In step loading, the deformation is instantaneous due to the Hookean spring; the initial state is Hertzian (i.e., purely elastic). The viscoelastic material behaves instantaneously as if the Kelvin-Voigt unit were not present. After an initial rapid increase, stabilization of the contact area occurs, with central contact pressure decreasing to accommodate the larger contact radius while maintaining a semi-ellipsoidal profile. Ramped loading exhibits ever-growing contact radii, but pressure appears to shift from semi-ellipsoidal to a uniform central plateau surrounded by a maximum. This is more apparent when the shear modulus of the Hookean spring is greater than that of the one in the Kelvin-Voight unit. Step – ramped down loading proves that load removal does not result in complete recovery of deformation as in the purely elastic case. These results are consistent with the general expectations regarding the viscoelasticity of the SLS, and encourage the use of the advanced MATLAB simulation tool in more complex contact scenarios.

Key words: viscoelastic contact, semi-analytical method, Standard Linear Solid (SLS) model.

1. INTRODUCTION

Modern materials such as polymers with long-chain macromolecular structures exhibit viscoelasticity, a unique behavior combining viscous and elastic deformation. Polymers can be tailored to be rigid, flexible, or elastomeric, allowing use in automotive (bumpers), aerospace (lightweight components), and medical fields (implants, syringes). Viscoelastic materials (e.g., in seat cushions, tires, and helmets) absorb shock and damp vibration, preventing noise and damage, while natural and synthetic viscoelastic polymeric substances are frequently integrated into synthetic synovial fluids designed for prosthetic articulations. Elucidating the mechanical response of such materials is crucial for design decisions, e.g., plastic components that must hold their shape over long periods but exhibit time-dependent performance due to creep and stress relaxation.
Theoretical viscoelasticity is a branch of continuum mechanics that describes materials exhibiting both viscous (time-dependent fluid-like) and elastic (solid-like) characteristics, such as polymers. The important concepts and modelling principles employed in this paper can be found in [1-3]. Contact mechanics [4,5] studies the deformation of solids that touch one another at one or more points. Combining viscoelasticity with contact

mechanics leads to complex mathematical models. The rare analytical closed form solutions advanced by Lee and Radok, by Hunter and subsequently by Ting, as cited in [4], exhibit restrictive assumptions that limit drastically their practical use in engineering applications. The existing theoretical developments lack versatility, especially when the load is described by non-monotonic functions of time, in which case the analytical development breaks down, and numerical solution of transcendental equations is needed. A general solution for the mechanical contact between solids of arbitrary boundary has not been achieved in linear elasticity, and its derivation becomes even more complicated for viscoelastic bodies due to the added time parameter.

Numerical simulation can advance the understanding of contact processes between viscoelastic bodies by reducing the need for expensive, time-consuming physical prototypes. It can predict, analyze, and optimize complex physical systems, providing insights into processes that are difficult or impossible to model mathematically. Simulating designs before physical production can significantly reduce development costs. Finite element analysis (FEA) is crucial for simulating structural, thermal, and fluid behavior in engineering designs, but may not be the optimal choice for contact mechanics. FEA can handle complex geometries, nonlinear material behavior, and complex boundary conditions, but the high gradients of stress and strain arising in concentrated contacts require very fine discretization. Moreover, the specifics of contact mechanics are that the deformation is confined to a vicinity of the initial point of contact, whereas the bulk of the body behaves like a rigid. This limits the versatility of FEA for contact mechanics simulations, and specific methods, derived from the boundary element method, have emerged [6]. These methods were first applied to contact simulations in elastic [7] and elastic-plastic domains [8,9], but can be extended to viscoelasticity [10,11] by considering all model parameters as functions of time and space. It is noteworthy that the replication of load histories was previously implemented for elastic-plastic contact simulations or fretting modeling. However, the approach implicitly neglected time-dependent parameters, primarily because, under such conditions, the material attributes do not vary with time, and the loading path can be replicated with the partition of load in small increments. As opposed to viscoelasticity, in which case, although the load may be kept constant, the contact parameters change due to the time-dependent material properties.

Based on the general algorithm presented in [11], this paper advances a computer code for the investigation of contact behavior of a Standard Linear Solid (SLS) half-space under various loading histories. Program validation is shown for step and ramped loading, while for ramped down loading, the results are new. The particularities of each loading path are explained considering the structure and features of the SLS model.

2. MATERIALS AND METHOD

Linear elasticity is often used as a constitutive equation for the description of stress-strain response in continuum mechanics. In this framework, a body subjected to an instantaneous load that is subsequently kept constant responds by instantaneous deformation, which remains constant and is fully and instantaneously recovered during unloading. This assumption may be acceptable for most metallic materials, but too limiting for other types of materials, e.g., polymers. The latter category tends to have a delayed mechanical response, whereas keeping the imposed stress or deformation constant does not guarantee a constant strain or stress response, respectively. Transient responses, in which the stress-strain response of the material depends explicitly upon time, can only be considered and modelled in a more general framework, i.e., that of viscoelasticity.

The latter is linked to concepts best described by simple uniaxial experiments: (a) material relaxation defines a process in which less stress is needed to preserve the same amount of strain, and (b) material creeps under constant stress, manifesting strain that increases with time. Regarding these experimental findings, the viscoelastic response can be mathematically described using two functions: (a) the relaxation modulus $\psi(t)=\sigma(t)/\varepsilon_0$, i.e., the ratio between time-dependent stress $\sigma(t)$ and a constant strain ε_0, and (b) the creep compliance function $\varphi(t)=\varepsilon(t)/\sigma_0$, i.e. the ratio of a time-dependent strain $\varepsilon(t)$ and a constant stress σ_0. Harmonic cyclic load requires a third function, namely the complex modulus, a material property that characterizes both the potential for strain energy storage and the viscous dissipation of internal energy within the viscoelastic continuum. The three functions are not independent, and the relation between them is the foundation of the linear viscoelasticity theory founded upon the Boltzmann superposition principle [1]: owing to the assumption of a linear system response, the cumulative effect of historical loading phases can be decoupled and analysed independently. The mathematical formulation of this principle is termed the Boltzmann-Volterra integral equations. By removing the time parameter from the latter equations, one gets Hooke's law. Thus, linear viscoelasticity is a generalization of linear elasticity.

Conceptual modelling of viscoelastic behaviour makes use of a combination of elastic (springs) and viscous (dashpots) components. Simpler models like Maxwell (a linear elastic spring and a viscous dashpot arranged in a series configuration) and Kelvin-Voigt (a purely elastic Hookean spring and a viscous damper connected in a parallel configuration) lack the capability for modelling creep, recovery, and stress relaxation at the same time. The simplest structure that accounts for these phenomena is obtained by arranging components in series and in parallel, as in Figure 1, and is referred to as the standard linear solid (SLS) model, or the Zener model.

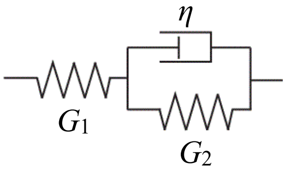

Fig. 1. SLS model, i.e. a Hookean spring and a Kelvin-Voigt element in series.

The following notations are used: $G_i, i=1,2$ are the spring stiffnesses, η is the dashpot viscosity, $\tau_i = \eta / G_2$ the retardation time, and $\tau_r = \eta/(G_1+G_2)$ the relaxation time. The SLS function $\varphi(t)$ is represented by the formula:

$$\varphi(t) = \frac{1}{2}\left\{\frac{1}{G_1} + \frac{1}{G_2}\left[1-\exp(-t/\tau_i)\right]\right\} = \frac{1}{2G_2}\left\{\frac{\tau_i}{\tau_i - \tau_r} - \exp(-t/\tau_i)\right\}, \tag{1}$$

and is applicable for quantifying deformation stemming from a history of effects, characterized by a time-varying pressure distribution. Only the normal (i.e., perpendicular to the body boundary) displacement u_z is considered in this paper, because the contact is assumed frictionless and therefore no tangential contact stresses arise. Based on Boltzmann's principle of superposition, the displacement response for linear viscoelastic materials is calculated as [10]:

$$u_z(x_1, x_2, t) = \int_0^{t_0}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varphi(t-t')B(x_1-x_1', x_2-x_2')\frac{\partial p(x_1', x_2', t')}{\partial t'} dx_1' dx_2' dt', \tag{2}$$

with *B* signifying the deformation of a semi-infinite elastic medium derived by Boussinesq, cited in [4]. The triple integral emerging from the linear viscoelastic framework of Boltzmann's superposition principle should be solved for arbitrary pressure history $p(x_1', x_2', t')$, which is only possible with numerical analysis. Discretization in space (i.e., of the common plane of contact) and time domains helps bring (2) to a computationally convenient form. Moreover, space discretization constitutes a requirement for the resolution of the contact simulation problem.
Apart from simulating material behaviour under load (i.e., the constitutive law), accurately reproducing the contact interaction necessitates the resolution of the underlying contact problem. In this paper, we are concerned with the co-called concentrated (or non-conforming) contact, in which the contacting surfaces have an initial point of contact (a geometrical idealisation) in the unloaded state. When a load is applied, the latter point grows into a contact area that is non-nil, but small by comparison with the important dimensions of the contacting bodies. This assumption allows presuming the contacting bodies as infinite flats in the contact vicinity. By choosing a so-called plane of contact that best splits the gap between the non-conforming boundaries, a "normal" direction is defined, and all pressure arising to counterbalance all resultant displacements acts parallel to the specified normal direction. Furthermore, by leveraging the theory of elastic half-spaces, fundamental solutions, specifically those proposed by Boussinesq and Cerruti, can be utilized to derive the necessary displacement components, as in equation (2). Whereas the contacting bodies are assumed as half-spaces in displacement computation, the opening between them is taken into consideration in the equation describing the geometry of the interfacial interaction mechanism:

$$h(x,y,t) = hi(x,y,0) + u_z(x,y,t) - \omega(t), \quad t \in [0, t_0]. \tag{3}$$

The latter synthesizes pre-loading metrics with values acquired during the interaction phase: $hi(x,y,0)$ is the opening between the contacting boundaries before loading, $u_z(x,y,t)$ is the two-dimensional sum of displacements, nil in the unloaded state but non-nil due to body deformation during loading, and $h(x,y,t)$ is the opening during load. The latter parameter vanishes over the contact area and serves as a constraint for residual reduction within the numerical resolution of the equation (3). The parameter $\omega(t)$ denotes the rigid-body approach, which initiates subsequent to loading, and evaluates the convergence of two nodes remote from the

contact region.
A two-dimensional Cartesian coordinate system is set at the interface layer. It is hypothesized that the solid bodies exhibit rigid behaviour in regions remote from the contact interface. The approach ω represents the cumulative linear deformation of both contacting bodies, oriented along the common normal through the initial point of contact.
Except $hi(x,y,0)$, everything else is unknown in equation (3). Moreover, in order to compute a current u_z, one needs the contact solution (pressure $p(x,y,t)$ and $\omega(t)$) in all previous states, i.e., the replication of the loading history. This additional obstacle is not present in linear elasticity and arises from the memory effect of the viscoelastic material shown by relation (2). Given all these intricate dependencies, an analytical solution with general applicability is difficult if not impossible, mainly because there exists no analytical solution to the contact problem in linear elasticity. It should be remembered that the notorious Hertz solution was derived by analogy with another physical process, while other solutions rely on the Hertz framework. Consequently, numerical analysis should be the method of choice, allowing substitution of the triple integration over space and time in the displacement equation (2) with multi-summation. The discrete counterparts of equations (2) and (3) are:

$$h(i,j,k) = hi(i,j,k) + u_z(i,j,k) - \omega(k)\,; \tag{4}$$

$$\begin{gathered} u_z(i,j,k) = \sum_{n=1}^{N_t}\sum_{\ell=1}^{N_1}\sum_{m=1}^{N_2} \varphi(k-n)G(i-\ell, j-m)\big(p(\ell,m,n) - p(\ell,m,n-1)\big), \\ i,\ell = 1\ldots N_1,\ j,m = 1\ldots N_2,\ k,n = 1\ldots N_t. \end{gathered} \tag{5}$$

It should be noted that discretization is applied in both time and space: a rectangular area around the initial point of contact is discretised with the aid of $N_1 \times N_2$ equally spaced grids, while the simulated time window $[0, t_0]$ is divided into N_t intervals. Relation (4) provides a means to express numerically the displacement $u_z(i,j,k)$ of the linear viscoelastic body in contact, provided all previous pressure distributions $p(\ell,m,n)$ are accounted for in each spatial control point. The multi-summation is robust but can become computationally intensive. For example, when N observation points are considered around the contact region, the contribution of pressure in each point to the displacement in each one should be computed and superimposed, thus an operation of order $O(N^2)$. To relieve the computational burden, the multi-summation is performed assisted by spectral methods, as described in [12,13]. This helps decrease the memory and CPU requirements and opens the method to fine meshes with 10^6 points, which are dense enough even for deterministic rough contact simulations.
Once the calculation of displacement in the grid nodes can be performed according to (5), the geometrical condition of deformation turns into a network of equations governing nodal pressure parameters. Excluding the rigid-body approach, the latter system exhibits linear behaviour with respect to pressure. Initial methodological approaches for such systems utilized an outer loop to facilitate iterative processing corresponding to the applied loading conditions. A formal analytical treatment regarding the contact model, incorporating the verification of existence and uniqueness of solutions, was provided in [14], whereas a survey of established methodologies is presented in [15].
The numerical method introduced by Polonsky and Keer [7] exploits the non-negativity of contact pressure over the contact area, incorporating this feature into an efficient strategy that utilizes the Conjugate Gradient solver for linear system resolution. Owing to the extensive magnitude of the system, direct methods like Gauss elimination are not optimal due to error accumulation. The linear system resulting from this must be solved for every discrete time and gives the current pressure. The latter is used, together with all previous pressure distributions attained during the loading history, to replicate a new state. The contact process results by assembly of all states computed for the chosen time moments. Regarding the temporal discretisation, its step should be chosen pertaining not only to the velocity of stress application but also concerning the retardation time of the viscoelastic material.
The algorithms outlined in this section were coded in MATLAB, resulting in software for the simulation of the mechanical contact involving a viscoelastic constitutive law. The input consists of: description of contact geometry before loading parameterized by spatial 2D coordinates, the material constitutive model (comprising viscoelastic properties, elastic moduli, and damping coefficients), the temporal evolution of the transmitted load, and the desired discretisation (number of spatial and temporal grids). The MATLAB script outputs: a 3D array of pressure, in which each slice comprises the pressure distribution for each temporal mark, a vector containing the rigid-body approach for each discrete time moment, and the vector of contact radii if the contact area is circular. The main strong points of the computer program are:

a) It is reasonably fast due to the calculation of multi-dimensional summations assisted by spectral methods: grids with 10^3 spatial control points and with 10^2 temporal control points are solved in minutes, while increasing the spatial density to 10^6 leads to hours of running time.
b) It can handle directly, without convergence issues, load histories that are non-monotonic functions of time. The program can also replicate the temporal contact behaviour after load removal.
The computer program was used to simulate the SLS viscoelastic contact under various types of loads, and the results are presented in the following section. Comparison with the literature and program validation are also achieved.

3. RESULTS AND DISCUSSIONS

Academic simulations of contact with nominal (i.e., generated by equations, not taking into account roughness) contact geometry are performed in this section. The *hi* parameter is numerically generated in the nodes of a 256 $\times 256$ mesh according to a sphere of radius $R = 0.018$ [m] indenting a half-space. For simplicity, the punch is assumed rigid while the half-space follows an SLS viscoelastic model with $G_1 = G_2 = 600$ [MPa] and $\eta = 3 \cdot 10^4$ [MPa $\cdot$ s], leading to a retardation time of $\tau_i = 50$ [s] and to a relaxation time of $\tau_r = 25$ [s]. In step loading, the load history $W(t)$ is generated with the aid of the Heaviside step function $H(t)$, so that $W(t) = W_{\max} H(t)$. Thus $W_{\max}$ is thus the maximum value of the force pressing the rigid ball against the viscoelastic half-space. The replicated temporal window of $10\tau_r$ is divided into 120 control points. For generality, the contact parameters achieved in instantaneous loading at $t = 0$, i.e. the contact radius $a(0)$, the central pressure $p_c(0)$, and the rigid-body approach $\omega(0)$, are employed as characteristic scales for non-dimensionalizing spatial coordinate, pressure and approach, respectively. The simulation running time on a 3.20 GHz six-core processor is under three minutes.
Utilizing the first case within Ting's theoretical framework as cited in [4], the pressure field distribution $p(r,t)$ can be articulated [16] as a function of both the radial coordinate r and the temporal variable t:

$$p(r,t) = \frac{4}{\pi R}\left\{2G_1\left[a^2(t) - r^2\right]^{\frac{1}{2}} - 2G_2\frac{(\tau_i - \tau_r)^2}{\tau_i \tau_r^2} e^{-\frac{t}{\tau_r}} \int_0^t e^{\frac{\xi}{\tau_r}}\left[a^2(\xi) - r^2\right]^{\frac{1}{2}} d\xi\right\}, \text{ with} \tag{6}$$

$$a(t) = \left\{\frac{3RW_{\max}}{16G_2}\left[\frac{\tau_i}{\tau_i - \tau_r} - e^{-\frac{t}{\tau_i}}\right]H(t)\right\}^{1/3}. \tag{7}$$

These two relations give similar numerical distributions as the MATLAB computer code. The data from relations (6) and (7) is displayed with continuous lines in figure 2, while the discrete symbols mark the predictions of the new MATLAB code. Given the problem complexity, the validation is excellent, although the two sets of data are not identical, but match within a few percent. The most important errors can be seen in the depiction of the contact radius. The latter variable exhibits step-wise, discrete fluctuations dictated by the spatial discretization scheme, wherein a rectangular cell of finite dimensions is characterized by binary inclusion or exclusion from the contact domain. This represents a fundamental limitation inherent to the numerical approach.
In ramped loading, a loading history $W(t) = W_{\max}\, m/\tau_i \cdot H(t)$, with m/τ_i the slope of the force increase, yields [16] a pressure distribution described by a relation (6) coupled with the following expression of the contact radius:

$$a(t) = \left\{\frac{3RW_{\max}}{16G_2}\frac{m}{\tau_i}\left[\frac{-\tau_i(\tau_i - \tau_r - t)}{\tau_i - \tau_r} + \tau_i e^{-\frac{t}{\tau_i}}\right]H(t)\right\}^{1/3}. \tag{8}$$

With respect to relations (6)-(8), it is imperative to acknowledge that: (a) pressure determination necessitates numerical approximation, and (b) the governing relations are valid exclusively under conditions of monotonic contact area expansion. The latter limitation does not exist with the MATLAB computer code. In Figure 3, the force was increased from 0 to $W_{\max} \cdot \tau_i = 5000$ [N] linearly with time in a temporal window $[0; \tau_i]$, while the other parameters were kept from the step loading simulation, except for the normalizers, which were substituted by their counterparts a_H, p_H and ω_H from the Hertz (purely elastic) contact featuring a contact radius *R*, a shear modulus G_2, a Poisson's ratio of 0.5, and loaded with $W_{\max}$.

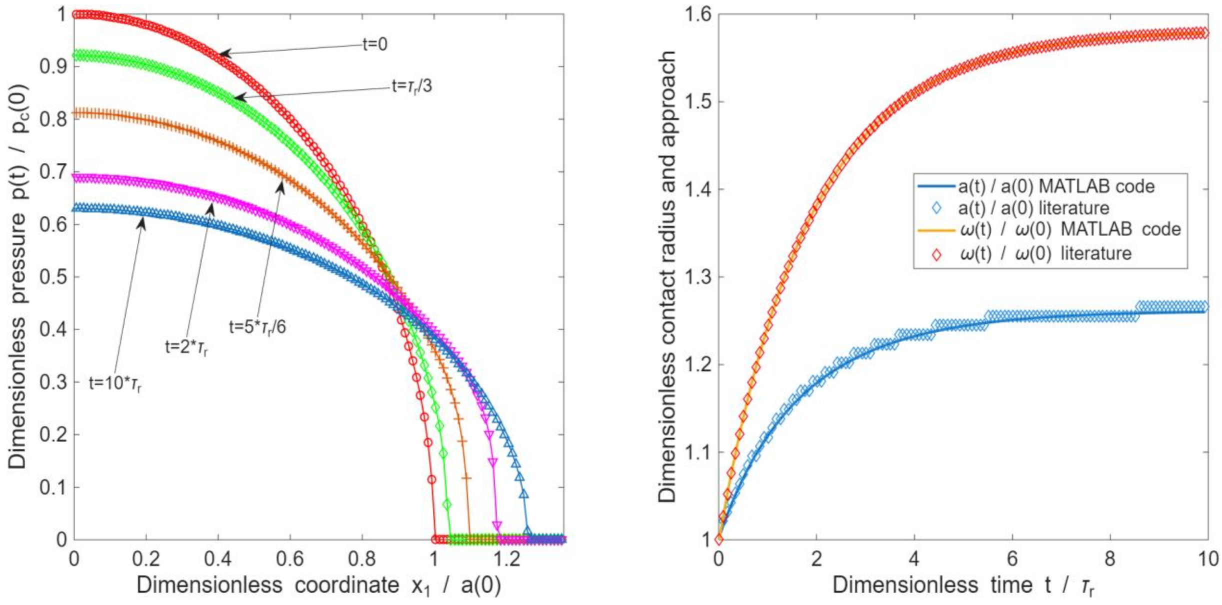

Fig. 2. Contact parameters in step loading: discrete symbols - MATLAB, solid lines - literature.

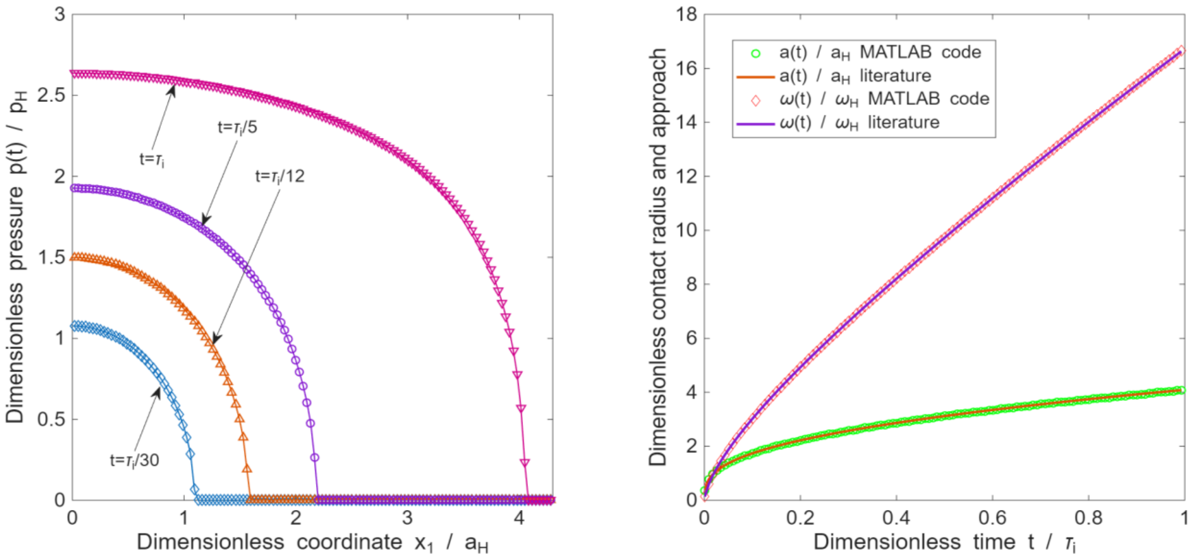

Fig. 3. Contact parameters in ramped loading, $G_1 = G_2$: discrete symbols - MATLAB, solid lines - literature.

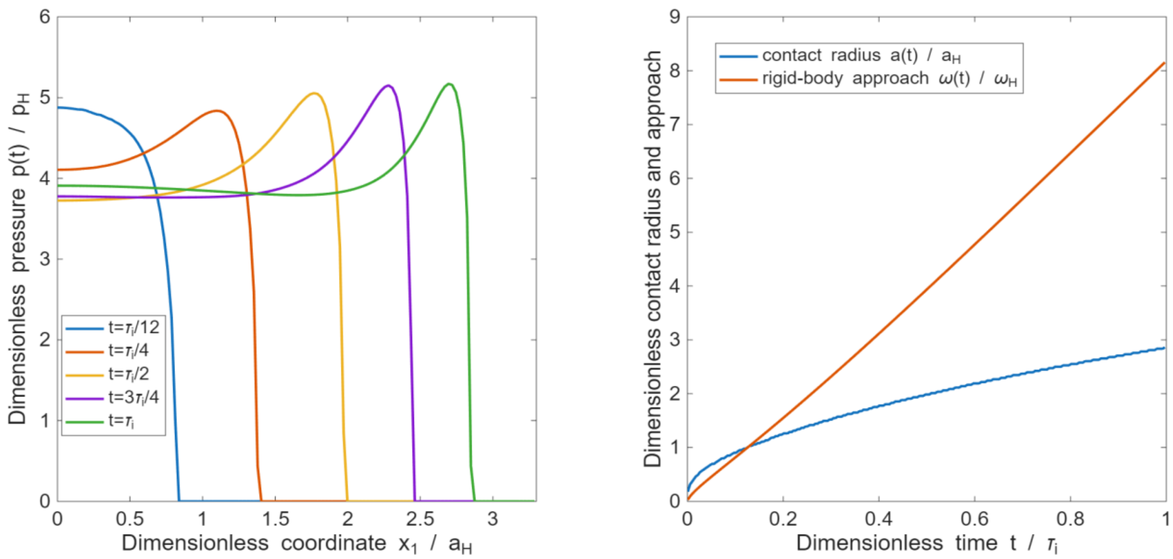

Fig. 4. MATLAB simulation of contact process in ramped loading, $G_1 = 10G_2$.

Significantly different pressure profiles are obtained by altering the ratio between the spring stiffnesses. In figure 4, all parameters were kept from the previous MATLAB simulation, except for G_1 that was chosen so that $G_1 = 10G_2$, giving a modified relaxation time of $\tau_r = 4.55$ [s].
In the last scenario, the load is ramped down from $W_{\max} \cdot \tau_i = 5000$ [N] to zero in a time window $[0; \tau_i]$, linearly with time. This case, in which the contact area does not increase monotonically, cannot be treated in the frame of Lee and Radok's framework, and leads to complex equations with Ting's cases, as cited in [4], whereas the new computer program does not require any additional modification. Simulation results are plotted in Figure 5, with the initial contact parameters $a(0)$, $p_{\max}(0)$ and $\omega(0)$ used as normalizers.

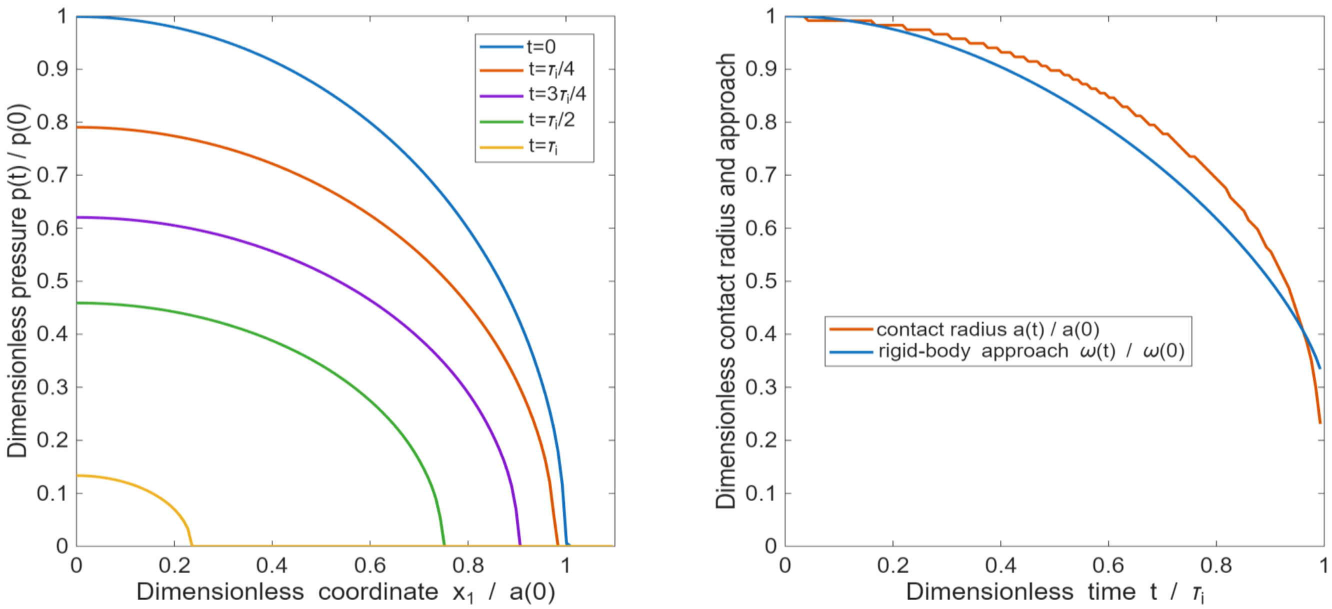

Fig. 5. MATLAB simulation of contact process in step - ramped down loading, $G_1 = G_2$.

4. CONCLUSIONS

The mechanical behaviour of modern materials, including polymers, cannot be modelled properly in the framework of linear elasticity. To account for time-dependent properties, viscoelastic models are needed. The Standard Linear Solid (SLS) framework, characterized by a Hookean elastic element coupled in series with a Kelvin-Voigt viscoelastic sub-unit, represents the foundational rheological model that can replicate creep, recovery, and stress relaxation.
Contact mechanics needs numerical analysis supported by spatial discretisation to achieve a solution for the contact problem between bodies of irregular boundary and having arbitrary but time-independent properties. Inclusion of viscoelasticity adds the discretisation in the temporal dimension. By using fundamental solutions for the half-space, a convenient technique for contact simulation was achieved, capable of replicating the loading path of contacting bodies following an SLS model. The advantage of this approach over finite element analysis is the confinement of the spatial discretisation to a two-dimensional vicinity of the initial point of contact, leading to a significant reduction of required computational resources.
A computer code for the simulation of contact processes with SLS behaviour was developed in MATLAB and tested for monotonic loading. The program was able to accurately reproduce results from the literature for step and ramped loading. Results for ramped loading with dissimilar springs and for ramped down loading were also presented.
In step loading, both contact radius and rigid-body approach exhibit instantaneous Hertzian value due to the Hookean spring, followed by subsequent stabilisation. The dashpot in the Kelvin-Voigt unit delays the deformation of its connected spring. The initial contact pressure distribution is Hertzian due to the Hookean spring, and with increasing time, keeps a semi-ellipsoidal shape but with increasing contact radius and decreasing central pressure.
Ramped loading shows contact radii and rigid-body approaches that increase with time, more rapidly at the beginning of loading. Pressure distribution has a semi-ellipsoidal shape that appears to flatten with increasing time. As opposed to step loading, where stabilisation is achieved, contact radius increases indefinitely to

accommodate the ever increasing load, but maximum pressure might shift toward the periphery, as demonstrated by the case with dissimilar springs.
Step - ramped down loading also exhibits an initial hertzian state and subsequent flattening of pressure on a decreasing contact area. At the end of the loading program, the contact radius and approach are not nil due to the dashpot viscosity of the Kelvin-Voigt unit, which delays the recovery of the initial form.
The advanced computer program appears to properly replicate various contact scenarios with monotonic load variation. This encourages its use in more complicated, non-monotonic loading programs, which is intended for future research efforts.

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